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16/ENG07/008
Petroleum Engineering
ENG 281

1a $\lim_{x \to \pi/2} \left[ \dfrac{\left(x^2 - \dfrac{\pi}{4}\right) \sin(\cos x)}{x - \pi/2} \right]$

Numerator

$u = x^2 - \pi/4 \qquad \dfrac{dy}{dx} = 2x$

$v = \sin(\cos x) \qquad \text{let } w = \cos x$

$v = \sin w \qquad \dfrac{dw}{dx} = -\sin x$

$\dfrac{dv}{dw} = \cos w$

$\dfrac{dv}{dx} = +\cos \cos x \cdot -\sin x$

$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{dy}{dx}$

$= (x^2 - \pi/4)(\cos \cos x \times -\sin x) + \sin(\cos x)\, 2x$

Denominator

$x - \pi/2$

$\dfrac{dy}{dx} = 1$

$\lim_{x \to \pi/2} = \left[ \left(\frac{\pi}{2}\right)^2 - \pi/4 \right] \left[ \cos(\cos \pi/2 x - \sin \pi/2) + \sin \cos \pi/2 \cdot \pi \right.$

$= \left[ \dfrac{\pi^2}{4} - \dfrac{\pi}{4} \right] \left[ \cos \cos 90 \times -\sin 90 \right] + \sin \cos 90 \times \pi$

$= \dfrac{\pi^2 - \pi}{4} \quad (-1) + (0)$

$= \dfrac{-\pi^2 - \pi}{4}$

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1b $\lim_{x \to \pi/2} \ln\left[\exp\left(\frac{3x^2 + 2x - 1}{x + 1}\right)\right]$

$= \ln\left[\exp \frac{3(\pi/2)^2 + 2(\pi/2) - 1}{\pi/2 + 1}\right]$

N.B $\ln(\exp) = 1$

$= \frac{\frac{3\pi^2}{4} + \frac{2\pi}{2} - 1}{\frac{\pi + 2}{2}} \qquad = \frac{3\pi^2 + 4\pi - 4}{4\ 2} \times \frac{2}{\pi + 2}$

$= \frac{3\pi^2 + 4\pi - 4}{2} \times \frac{1}{\pi + 2}$

$= \frac{\cancel{8}\cdot(3\pi - 2)\cancel{(\pi + 2)}}{2} \times \frac{1}{\cancel{\pi + 2}}$

$= \frac{3\pi - 2}{2}$

1c. $\lim_{x \to 2 + \sqrt{3}} \cos\left[\sin^{-1} \frac{(x - 2)}{(x - \sqrt{3})}\right]$

$= \cos\left[\sin^{-1} \frac{2 + \sqrt{3} - 2}{2 + \sqrt{3} - \sqrt{3}}\right]$

$= \cos\left[\sin^{-1} \frac{\sqrt{3}}{2}\right]$

$= \cos 60$

$= 0.5$

d. $\lim_{x \to 4} \left[\frac{x^2 - 8x + 16}{x^2 - 5x + 4}\right]$

$\lim_{x \to 4} \left[\frac{2x - 8}{2x - 5}\right]$

$= \frac{2(4) - 8}{2(4) - 5} = \frac{8 - 8}{8 - 5} = \frac{0}{3} = 0$

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2. Determine whether its convergent

a, $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \cdots$

From standard series

$\frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p} + \frac{2}{5^p} + \cdots$

When p is 2

$\frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \frac{2}{5^2} + \cdots$

Since $\frac{2}{2\times3} < \frac{2}{2\times2}$, $\frac{2}{3\times4} < \frac{2}{3\times3}$

Therefore the series is convergent

b, $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

For comparison test using standard series when $p=2$

$\frac{2}{1^p} + \frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p} + \cdots$

Since $P>1$, the series converges

Therefore the series is convergent

c, $U_n = \frac{1+2n^2}{1+n}$

$\lim_{n\to\alpha} U_n = \lim_{n\to\alpha} \left[\frac{1/n^2 + 2n^2/n^2}{1/n^2 + n^2/n^2}\right]$

$= \left[\frac{1/n^2 + 2/1}{1/n^2 + 1/1}\right] = \frac{0+2}{0+1} = \frac{2}{1} = 2$

Since $\lim_{n\to\alpha} U_n \neq 0$

the series is not convergent

3 $U_n = \dfrac{x^n}{(2n+1)^3}$

$U_{n+1} = \dfrac{x^{n+1}}{(2n+2)^3}$

$\dfrac{U_{n+1}}{U_n} = \dfrac{x^n \cdot x}{(2n+2)^3} \times \dfrac{(2n+1)^3}{x^n}$

$= \dfrac{x\,(4n^2+4n+1)(2n+1)}{(4n^2+8n+4)(2n+3}$

$= \dfrac{x\,(8n^3+8n^2+2n+4n^2+4n+1)}{8n^3+16n^2+8n+8n^2+16n+8}$

$= \dfrac{x\,(8n^3+12n^2+6n+1)}{8n^3+24n^2+24n+8}$

$= \lim\limits_{n\to\infty} \dfrac{8n^3/n^3 + 12n^2/n^3 + 6n/n^3 + 1/n^3}{8n^3/n^3 + 24n^2/n^3 + 24n/n^3 + 8/n^3}$

$= x\dfrac{(8+0+0+0)}{(8+0+0+0)}$

$= x\dfrac{\cancel{8}}{\cancel{8}}$

$\lim\limits_{n\to\infty} \left|\dfrac{U_{n+1}}{U_n}\right| = x$

For absolute convergence $\lim\limits_{n\to\infty} \left|\dfrac{U_{n+1}}{U_n}\right| < 1$

$\therefore -\leq x \leq 1$ is when $x$ is converges

4 Evaluate using L'Hopital's rule

$$\lim_{x \to 0} \left[ \frac{\sin x - \cos x}{x^3} \right]$$

$$= \lim_{x \to 0} \left[ \frac{\cos x + \sin x}{3x^2} \right]$$

$$= \lim_{x \to 0} \left[ \frac{-\sin x + \cos x}{6x} \right]$$

$$= \lim_{x \to 0} \left[ \frac{-\cos x + (-\sin x)}{6} \right]$$

$$= \frac{-\cos 0 - \sin 0}{6}$$

$$= \frac{-1 + 0}{6} = \frac{-1}{6}$$